

Asymptotic behavior of ω in general quintom model

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Abstract

For the quintom models with arbitrary potential $V = V(\varphi, \sigma)$, the asymptotic value of equation of state parameter ω is obtained by a new method. In this method, ω of stable attractors are calculated by using the ratio $d \ln V / d \ln a$ in asymptotic region. All the known results, have been obtained by other methods, are reproduced by this method as specific examples.

1 Introduction

The recent observations on type Ia supernova [1], large scale structure [2] and cosmic microwave background radiation [3], support the claim that our universe has accelerating expansion and it is composed of dark energy (73%), dark matter (23%) and baryonic matter (4%).

The simplest candidate for dark energy is a cosmological constant Λ of order $(10^{-3} \text{ev})^4$, but it suffers from conceptual problems such as fine-tuning and coincidence problems [4]. As the alternative to cosmological constant, the dynamical models have been introduced.

One of the important parameters of dark energy is the equation of state parameter $\omega = p/\rho$, which must satisfy $\omega < -1/3$ if one interested in accelerating universe. The time dependence of ω , i.e. $\omega(z)$, is usually used to explain the dynamical nature of our universe. Some astrophysical data seems to slightly favor an evolving dark energy and shows a recent $\omega = -1$, the so-called phantom-divide-line, crossing [5].

Many of dynamical dark energy models are based on scalar fields. The simplest of these models is the quintessence model which consists of one normal scalar field φ [6]. In all quintessence models, always $\omega > -1$. The other kind of scalar field model is phantom model. In this model, there exists a phantom scalar field σ which its kinetic term appears with minus sign. In phantom model, ω always satisfies $\omega < -1$ [7]. None of these two models can explain the $\omega = -1$ crossing and at least two scalar fields are needed, in models known as hybrid models, for occurrence of this transition. One of these models is the quintom model which consists of one quintessence and one phantom fields [8]. One can show that in all quintom models with slowly-varying potentials, the transition

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from $\omega > -1$ to $\omega < -1$ is always possible [9]. Related to $\omega = -1$ crossing, it must be added that the occurrence of this transition is also possible in a model with only one scalar field, but this field must be coupled to background matter field [10].

One of the interesting topics in dark energy problem is the studying of the late-time, or asymptotic, behavior of physical quantities. In this connection, the investigation of attractors of the various dark energy models is one of the main tools [11]. Among the attractors of a dynamical system, the stable attractors determine the late-time behavior of various quantities, including the equation of state parameter ω . Despite its usefulness, the attractors' studies are restricted to specific potentials. This is because the choosing of the set of convenient variables, which results in an autonomous set of equations, is a highly nontrivial task and depends on the functional form of the considered potential. Therefore seeking other methods for studying the late-time behavior of, for instance, ω is an important task.

In ref.[12], the asymptotic behavior of ω_σ of phantom models has been investigated for potentials which $V(\sigma) \rightarrow \infty$ asymptotically. There, it has been shown that all such models are divided into three classes, characterized by $\omega_\sigma \rightarrow -1$, $\omega_\sigma \rightarrow \omega_0 < -1$ and $\omega_\sigma \rightarrow -\infty$. In ref.[13], the evolution of a special quintom model, the so-called hessence model, has been discussed in $\omega - \omega'$ plane. The hessence model is a quintom model with potential $V(\varphi, \sigma) = V(\varphi^2 - \sigma^2)$. φ and σ denote the quintessence and phantom fields, respectively. In this model, there is, effectively, only one dynamical field $\phi = \sqrt{\varphi^2 - \sigma^2}$. The late time behavior of this model has been classified in $\omega - \omega'$ plane in [13] and it has been shown that there exist four distinct regions in this plane. Both of the above mentioned studies are restricted to cases with only one dynamical dark energy field.

In this paper we want to study the asymptotic behavior of equation of state parameter ω for quintom models with arbitrary potential. Since it is a two-field-component model, there exists new features which do not appear in single-component models of [12] and [13]. It is must be added that the attractors of quintom models with two classes of potentials has been recently studied in [14]. The potentials are restricted to cases in which the scalar fields do not diverge, and the potentials which behave asymptotically as an exponential potential. As we will see, there are too many examples that do not belong to the above mentioned cases, (to be specific, the examples B, C, D, F and G of section 2), and one must seek another method, as we do, to obtain the asymptotic behavior of the system.

The paper is organized as follows. In section two, we briefly review the quintom model and derive a relation which describes the dynamical evolution of the quintom potential. This equation is one that we use to obtain the asymptotic behavior of equation of state parameter ω of an arbitrary quintom model. It is the same relation which exists in hessence model [13], and reduces to the known relations in quintessence [15] and phantom [16] models in appropriate limits. In section three, we use this relation to obtain the asymptotic value of ω for several examples. The examples are divided to two classes: the potentials which have no quintessence-phantom interaction, i.e. $V(\varphi, \sigma) = V_1(\varphi) + V_2(\sigma)$, and the potentials containing the interaction term $V_{\text{int}}(\varphi, \sigma)$. In all examples, it is seen that our results coincide with those obtained by other methods, e.g. the attractors' study or the numerical study of fields' equations.

We use the units $c = \hbar = 8\pi G = 1$ throughout the paper.

2 The evolution equation of quintom potential

we consider a spatially flat Friedman-Robertson-Walker space-time in comoving coordinates (t, x, y, z)

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \quad (1)$$

where $a(t)$ is the scale factor. The quintom dark energy consists of the quintessence field φ and the phantom field σ , with Lagrangian

$$\mathcal{L}_{\text{de}} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\varphi, \sigma). \quad (2)$$

The energy density ρ_{de} and pressure p_{de} of the homogenous quintom dark energy are

$$\begin{aligned} \rho_{\text{de}} &= \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}\dot{\sigma}^2 + V(\varphi, \sigma), \\ p_{\text{de}} &= \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}\dot{\sigma}^2 - V(\varphi, \sigma), \end{aligned} \quad (3)$$

and the evolution equations of the fields are

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial V(\varphi, \sigma)}{\partial \varphi} = 0, \quad (4)$$

and

$$\ddot{\sigma} + 3H\dot{\sigma} - \frac{\partial V(\varphi, \sigma)}{\partial \sigma} = 0. \quad (5)$$

Here $H(t) = \dot{a}(t)/a(t)$ is the Hubble parameter and 'dot' denotes the time derivative. The Friedman equations are

$$3H^2 = \rho_{\text{de}}, \quad (6)$$

and

$$-2\dot{H} = \rho_{\text{de}} + p_{\text{de}}. \quad (7)$$

Note that the equations (4)-(7) are not independent and eq.(7) is obtained from eqs.(4)-(6). The equation of state parameter $\omega = p_{\text{de}}/\rho_{\text{de}}$ is therefore

$$\omega = \frac{\dot{\varphi}^2 - \dot{\sigma}^2 - 2V(\varphi, \sigma)}{\dot{\varphi}^2 - \dot{\sigma}^2 + 2V(\varphi, \sigma)}. \quad (8)$$

Because of the field equations (4) and (5), the quintessence field φ rolls down the potential to reach a minimum of V , while the phantom field σ falls up the potential V and settles in a maximum of V . So the late-time values of (φ, σ) are the saddle point of potential V , we denote it by (φ^*, σ^*) . We will use this fact in our analysis.

To obtain the evolution equation of potential V , we first define the function

$$x = \left| \frac{1 + \omega}{1 - \omega} \right| = \frac{\pm(\dot{\varphi}^2 - \dot{\sigma}^2)}{2V}, \quad (9)$$

if $\omega \neq -1$. Differentiating $\ln x$ with respect to $\ln a$ results in

$$\frac{d \ln x}{d \ln a} = \frac{1}{H} \left\{ \frac{2(\dot{\varphi}\ddot{\varphi} - \dot{\sigma}\ddot{\sigma})}{\dot{\varphi}^2 - \dot{\sigma}^2} - \frac{\dot{V}}{V} \right\}. \quad (10)$$

Using eqs.(4) and (5) and noting that $\dot{V} = (\partial V/\partial \varphi)\dot{\varphi} + (\partial V/\partial \sigma)\dot{\sigma}$, eq.(10) can be written as

$$\frac{d \ln x}{d \ln a} = \frac{1}{H} \left\{ -6H - \frac{2}{1+\omega} \frac{\dot{V}}{V} \right\}, \quad (11)$$

or

$$-\frac{\dot{V}}{V} = 3H(1+\omega) \left(1 + \frac{1}{6} \frac{d \ln x}{d \ln a} \right). \quad (12)$$

This is a general equation which can be used for $\varphi = 0$ case (which the quintom model becomes phantom model), $\sigma = 0$ case (where the model is the quintessence model), and when $V(\varphi, \sigma) = V(\phi = \sqrt{\varphi^2 - \sigma^2})$, which the quintom model becomes hessence model. In all above mentioned cases, the potential is a function of only one field variable, e.g. the field θ , and therefore eq.(12) can be reduced to an equation for the field-derivative of potential. This is because $\dot{V} = (dV/d\theta)\dot{\theta}$. But in the quintom model, it is not the case.

It is more suitable to write the left-hand-side of eq.(12) in terms of derivative with respect to $\ln a$. The result is

$$-\frac{V'}{V} = 3(1+\omega) \left(1 + \frac{1}{6} \frac{x'}{x} \right), \quad (13)$$

in which

$$f' := \frac{df}{d \ln a} = \frac{1}{H} \dot{f}. \quad (14)$$

We are going to use eq.(13) to determine the late-time behavior of ω for quintom models, i.e. to obtain the value ω_0 for stable attractors of quintom models. To do so, we must first evaluate the ratio x'/x for attractor solutions. The first step in studying the attractors is to introduce a set of convenient dimensionless variables, which in study of quintom models, three of these variables are

$$x_\varphi = \frac{\dot{\varphi}}{\sqrt{6}H}, \quad x_\sigma = \frac{\dot{\sigma}}{\sqrt{6}H}, \quad y = \frac{\sqrt{V}}{\sqrt{3}H}. \quad (15)$$

The other variables depend on the functional form of the potential, but the above three variables always exist. In terms of these variables, the ratio x'/x becomes

$$\frac{x'}{x} = \frac{2\omega'}{1-\omega^2} = \frac{2[x_\varphi y^2 x'_\varphi - x_\sigma y^2 x'_\sigma + (x_\sigma^2 - x_\varphi^2) y y']}{y^2(x_\varphi^2 - x_\sigma^2)}. \quad (16)$$

For attractor solutions, one has $x'_\varphi = x'_\sigma = y' = 0$, so $x'/x \rightarrow 0$. Of course one must be more careful for situation in which $y^2(x_\varphi^2 - x_\sigma^2) = 0$. In quintom models, the fields φ and σ satisfy different equations of motion, i.e. φ moves toward the minimum of potential while σ goes to the maximum of V . Therefore we expect that in stable attractor solutions, $y^2 \sim V(\varphi^*, \sigma^*) \neq 0$ and usually $x_\varphi^2 \neq x_\sigma^2$. In next section, we see that these conditions are fulfilled in all known

examples, except the example F which we discuss it separately. So for stable attractors, the equation of state parameter ω tends to ω_0 , obtained from

$$\omega \rightarrow \omega_0 = -1 - \frac{1}{3} \frac{V'}{V}. \quad (17)$$

3 Examples

In this section we apply the main result (17) to situations which have been studied by other methods. These examples are divided to two classes: The quintom models without quintessence-phantom interaction and the potentials contain this interaction term. We discuss these two kinds of potential separately.

3.1 The potentials $V(\varphi, \sigma) = V_1(\varphi) + V_2(\sigma)$

For potentials of the form $V(\varphi, \sigma) = V_1(\varphi) + V_2(\sigma)$, the asymptotic configurations of the fields φ and σ are (φ^*, σ^*) , where $V_1(\varphi)$ becomes minimum at φ^* and $V_2(\sigma)$ attains its maximum at σ^* . In following we consider four examples.

A. $V = V_{\varphi_0} e^{-\lambda_1 \varphi} + V_{\sigma_0} e^{-\lambda_2 \sigma}$

The attractors of this potential have been studied in [17]. As the exponential function has no extremum points, so we can only use the fact that φ always rolls down the potential and σ falls up it. So $\varphi^* \rightarrow \infty$ and $\sigma^* \rightarrow -\infty$, from which $V(\varphi, \sigma) \rightarrow V_2(\sigma)$. The ratio V'/V then becomes

$$\frac{V'}{V} \simeq \frac{V'_2}{V_2} = \frac{(\partial V_2 / \partial \sigma) \sigma'}{V_2} = -\lambda_2 \sigma'. \quad (18)$$

Eq.(17) then results in

$$\omega \rightarrow \omega_0 = -1 + \frac{1}{3} \lambda_2 \sigma'. \quad (19)$$

This is exactly the result obtained in [17]. The only stable attractor of this potential is characterized by $x_\varphi = 0$, $x_\sigma = -\lambda_2 / \sqrt{6}$ and $y^2 = 1 + \lambda_2^2 / 6$. So

$$\omega = \frac{x_\varphi^2 - x_\sigma^2 - y^2}{x_\varphi^2 - x_\sigma^2 + y^2} = -1 - \frac{\lambda_2^2}{3}, \quad (20)$$

which is equal to (19) (note that $x_\sigma = \sigma' / \sqrt{6} = -\lambda_2 / \sqrt{6}$, results in $\sigma' = -\lambda_2$, so $\omega = -1 - \lambda_2^2 / 3 = -1 + (1/3) \lambda_2 \sigma'$).

B. $V = V_{\varphi_0} \varphi^\alpha + V_{\sigma_0} \sigma^\alpha$ ($\alpha > 0$)

The time-variation of ω of this potential has been studied for two special cases $\alpha = 2$ and $\alpha = 1.8$ in [18]. $V_1(\varphi)$ is minimum at $\varphi^* = 0$, but $V_2(\sigma)$ has no maximum, although it is clear that $|\sigma^*| \rightarrow \infty$. So $V(\varphi, \sigma) \rightarrow V_2(\sigma)$ and

$$\frac{V'}{V} \simeq \frac{V'_2}{V_2} = \frac{2\sigma'}{\sigma} \Big|_{\sigma \rightarrow \pm\infty} \longrightarrow 0, \quad (21)$$

from which eq.(17) results in

$$\omega \rightarrow \omega_0 = -1. \quad (22)$$

This is the same result obtained for special cases in [18]. It can be shown that eq.(22) is also valid for odd α 's where $\varphi^* \rightarrow -\infty$.

C. $V = V_{\varphi_0} e^{-\lambda_1 \varphi^2} + V_{\sigma_0} e^{-\lambda_2 \sigma^2}$

$V_1(\varphi)$ has no minimum, but it is obvious that $|\varphi^*| \rightarrow \infty$. $V_2(\sigma)$ is maximum at $\sigma^* = 0$. So again $V(\varphi, \sigma) \rightarrow V_2(\sigma)$, from which

$$\frac{V'}{V} \simeq \frac{V'_2}{V_2} = -2\lambda_2 \sigma \sigma' |_{\sigma=0} = 0. \quad (23)$$

This results in $\omega_0 = -1$ which is consistent with numerical calculation of [17].

D. $V = V_0 e^{\lambda \sigma^\alpha}$ ($\alpha > 0$)

Now we consider a phantom model with the above potential. In this case $|\sigma^*| \rightarrow \infty$. Therefore

$$\frac{V'}{V} = \alpha \lambda \sigma^{\alpha-1} \sigma' |_{\sigma=\pm\infty} = \begin{cases} 0, & \alpha < 1 \\ \lambda \sigma', & \alpha = 1, \\ \infty, & \alpha > 1 \end{cases} \quad (24)$$

from which

$$\omega_0 = \begin{cases} -1, & \alpha < 1 \\ -1 - \lambda \sigma' / 3, & \alpha = 1. \\ -\infty, & \alpha > 1 \end{cases} \quad (25)$$

In above we use the fact that in α =even cases, we have $\sigma^* \rightarrow \pm\infty$. For $\sigma^* \rightarrow +\infty$, since $\sigma' > 0$, one has $\sigma^{\alpha-1} \sigma' \rightarrow +\infty$. For $\sigma^* \rightarrow -\infty$, we know that $\sigma' < 0$ and therefore again $\sigma^{\alpha-1} \sigma' \rightarrow +\infty$. This general result agrees with the numerical calculations of [12] which have been done for special cases $\alpha = 0.5, 1$, and 2 with $\lambda = 1$. It is clear that eq.(25) can be also used as the late-time behavior of ω of a quintom model with the potential $V = V_{\varphi_0} e^{\lambda_1 \varphi^\alpha} + V_{\sigma_0} e^{\lambda_2 \sigma^\alpha}$.

3.2 The interacting potentials

We now consider the potentials of the form $V(\varphi, \sigma) = V_1(\varphi) + V_2(\sigma) + V_{\text{int.}}(\varphi, \sigma)$, through three following examples.

E. $V = V_0 e^{-\sqrt{6}(m\varphi+n\sigma)}$

The attractors of this quintom potential have been discussed in [19]. This potential has no extremum point, but it is clear that $\varphi^* \rightarrow \infty$ and $\sigma^* \rightarrow -\infty$. The ratio V'/V is

$$\frac{V'}{V} = \frac{1}{V} \left(\frac{\partial V}{\partial \varphi} \varphi' + \frac{\partial V}{\partial \sigma} \sigma' \right) = -\sqrt{6}(m\varphi' + n\sigma'), \quad (26)$$

so

$$\begin{aligned} \omega_0 &= -1 + \frac{\sqrt{6}}{3}(m\varphi' + n\sigma') \\ &= -1 + 2(mx_\varphi + nx_\sigma). \end{aligned} \quad (27)$$

In [19], it has been found two stable attractors, denoted there by P and T solutions. In P-case, the coordinates are $x_\varphi = m$, $x_\sigma = -n$ and $y^2 = 1 - m^2 + n^2 \neq 0$, which result in $\omega_P = -1 + 2(m^2 - n^2)$. This is the same as our result (27). In T-case, it has been obtained $x_\varphi = m/(2(m^2 - n^2))$, $x_\sigma = -n/(2(m^2 - n^2))$ and $y^2 = 1/(4(m^2 - n^2))$, with $\omega_T = 0$. This solution is also obtained from (27). It is interesting that the ω of other solutions of [19] are not reproduced by using (27). It is because these solutions are unstable attractors and therefore do not describe the late-time behavior of the system.

F. $V = \frac{1}{2}m^2\varphi^2 + \frac{1}{2}g^2\varphi^2\sigma^2 + (M^2 - \lambda\sigma^2)^2/(4\lambda)$

This potential has been investigated in [14]. In this case, it can be easily seen that the potential has a unique saddle point at $(\varphi^*, \sigma^*) = (0, 0)$. Therefore

$$\frac{V'}{V} = \frac{1}{V} \left(\frac{\partial V}{\partial \varphi} \varphi' + \frac{\partial V}{\partial \sigma} \sigma' \right)_{(\varphi, \sigma) = (0, 0)} = 0, \quad (28)$$

which results in $\omega_0 = -1$. This is the same value obtained in [14], denoted there by de Sitter solution. Note that here $x_\varphi = x_\sigma = 0$ and $y = 1$. So eq.(16) leads to

$$\frac{x'}{x} \rightarrow -\frac{2y'}{y}, \quad (29)$$

which again becomes zero at $y' = 0$ and $y = 1$.

G. $V = V_1 + V_2 + \lambda\sqrt{V_1 V_2}$, **where** $V_1 = V_{\varphi_0} e^{-\lambda_1 \varphi}$ **and** $V_2 = V_{\sigma_0} e^{-\lambda_2 \sigma}$

This is the potential whose attractors have been obtained in [20]. This potential has no extremum point, but it is clear that $\varphi^* \rightarrow \infty$ and $\sigma^* \rightarrow -\infty$. In this case

$$\begin{aligned} \frac{V'}{V} &= \frac{1}{V} \left[-\lambda_1 (V_1 + \frac{\lambda}{2} \sqrt{V_1 V_2}) \varphi' - \lambda_2 (V_2 + \frac{\lambda}{2} \sqrt{V_1 V_2}) \sigma' \right]_{(V_1, V_2) \rightarrow (0, \infty)} \\ &= -\lambda_2 \sigma'. \end{aligned} \quad (30)$$

So

$$\omega_0 = -1 + \frac{1}{3} \lambda_2 \sigma'. \quad (31)$$

The only stable attractor of this potential is the second solution of Table 1 of [20] with coordinates $x_\varphi = 0$, $x_\sigma = -\lambda_2/\sqrt{6}$ and $y^2 = 1 + \lambda_2^2/6$. These values result in $\omega = -1 - \lambda_2^2/3 = -1 + \lambda_2 \sigma'/3$, which is same as eq.(31).

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